Day 19 - Power and Types of Error

Learning outcomes

In this section, we briefly step away from inference to discuss the ways in which we can be wrong.

Warm-up

Suppose I conduct a hypothesis test and obtain a small p-value, and thus reject the null hypothesis. What error could I have made?

Rejected H_0 when it is actually true.

Suppose I conduct a hypothesis test and obtain a large p-value, and thus fail to reject the null hypothesis. What error could I have made?

It could actually be false.

Power and types of error

$H_0: \pi = .5$

$H_a: \pi \neq .5$

known

Truth \ Decision	Reject H_0	Fail to reject H_0
H_0 true ex. ($\pi = .5$)	Type I error	☺
H_0 false ($\pi = .47$, $\pi = .75$, ⋮)	☺ Probability of being in this box is Power	Type II error

Type I example: Saying there is a difference in the way Coke & Pepsi taste when in fact there is not.

Type II example: (1) Saying there is no evidence that we can differentiate Coke & Pepsi, when in fact we can.

(2) Concluding there is no evidence that the probability of identifying Coke is greater than 0.5,

Understanding power and errors

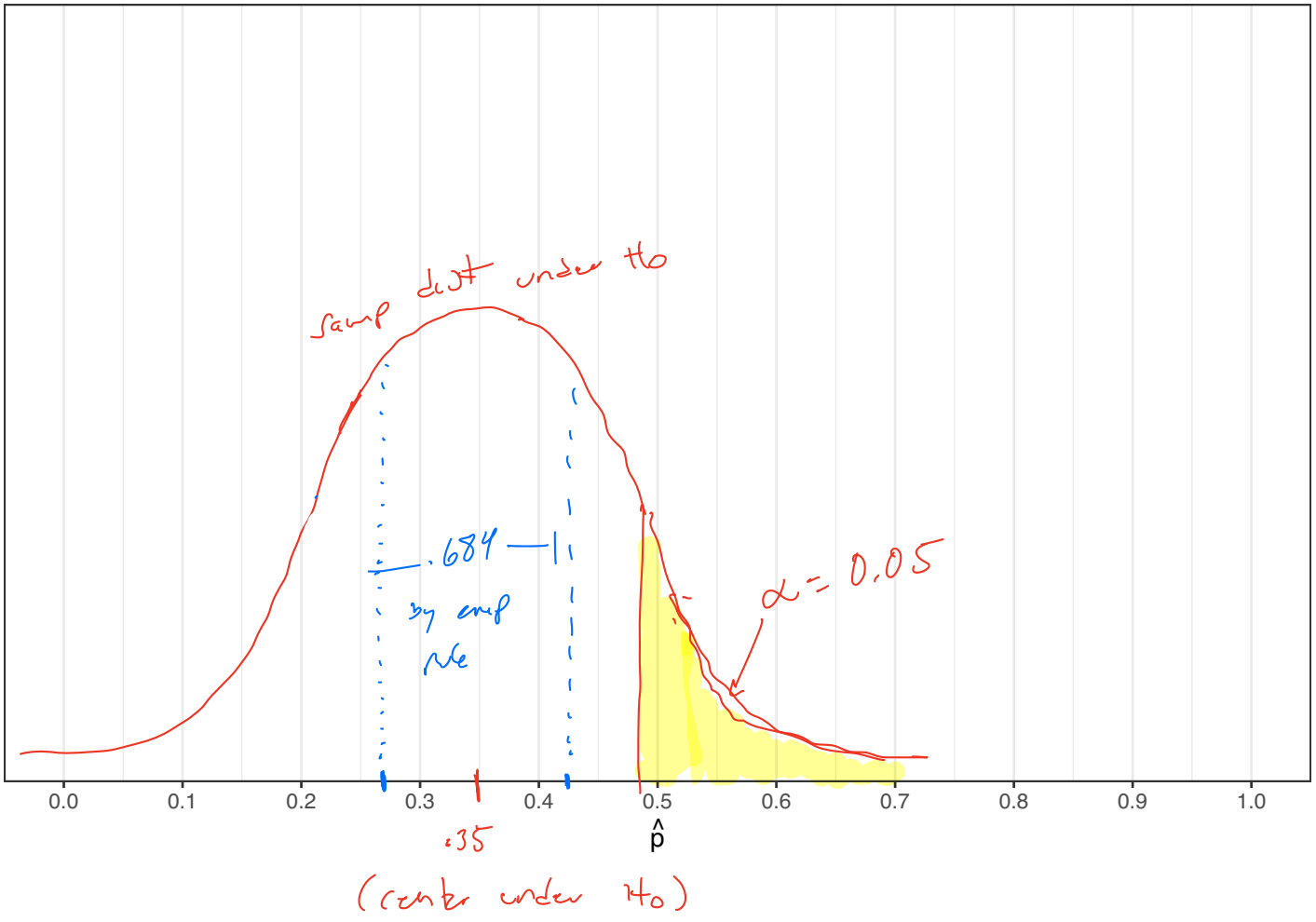
Consider a fixed-level hypothesis test of

$$H_0 : \pi = 0.35$$
$$H_a : \pi > 0.35$$

with $\alpha = 0.05$. If the **true population proportion is actually 0.55, what is the probability that we reject the null hypothesis if we collect a sample of size 40?**

To answer this question, let's first consider two sampling distributions of the sample proportion: the one assuming H_0 is true, and the actual sampling distribution from the true value of π.

Sampling distributions of $\hat{p}$

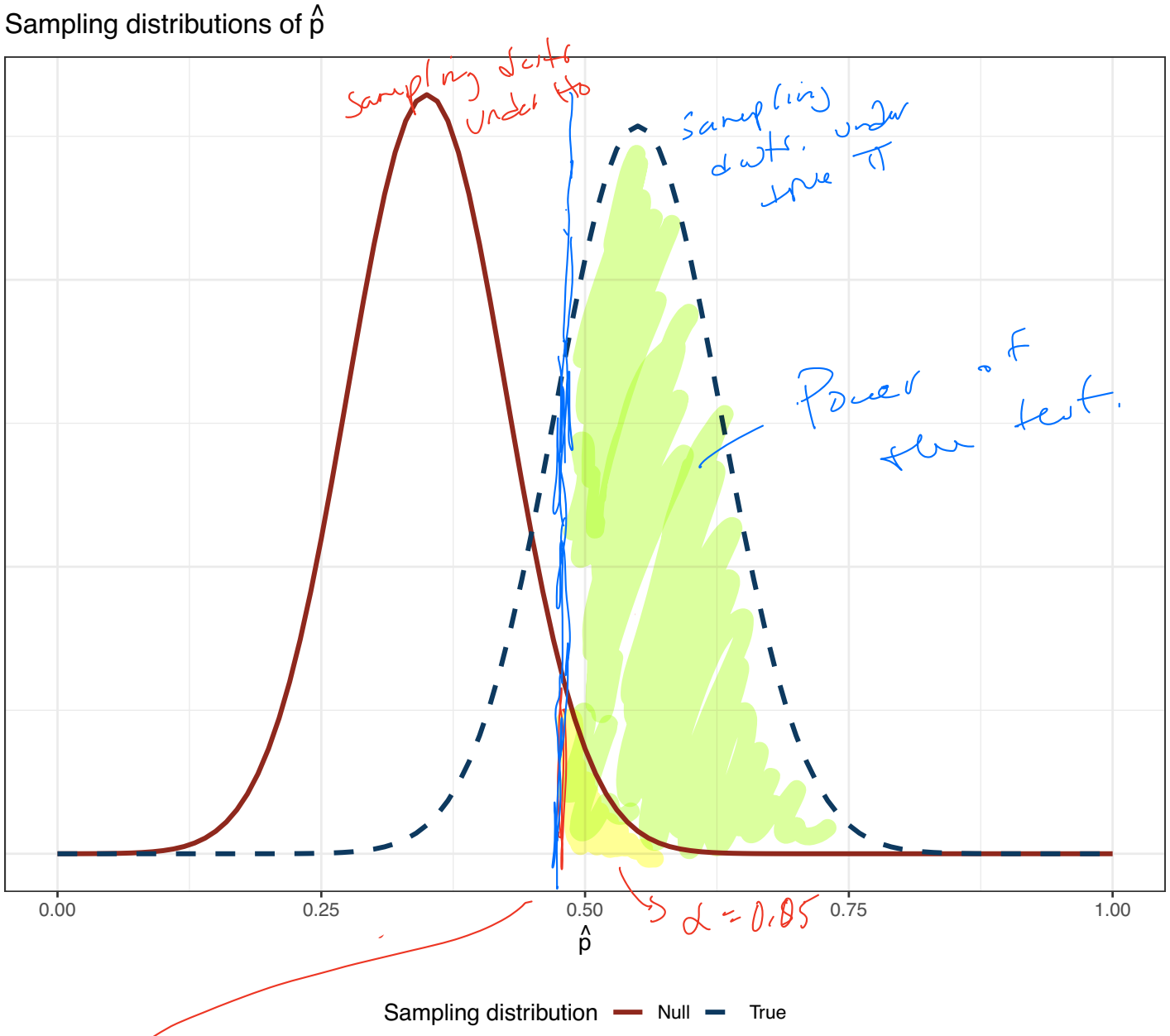


Under H_0:

$$SD(\hat{p}) = \sqrt{\frac{\pi_0(1-\pi_0)}{n}} = \sqrt{\frac{.35(.65)}{40}} = 0.075$$

Draw a vertical line on the plot at the threshold that we would use to reject the null hypothesis. We want the value k such that:

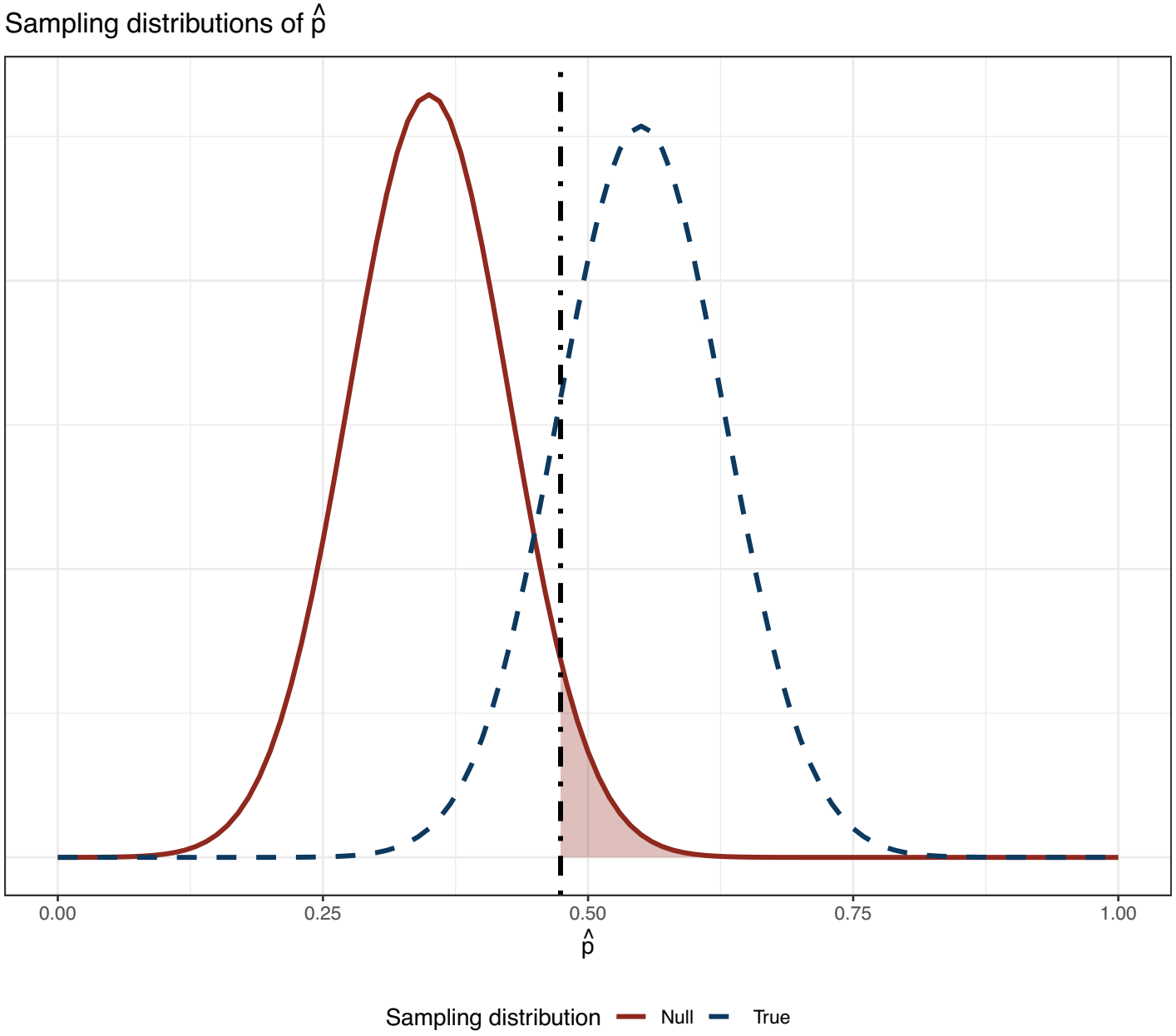
$$P(\hat{p} > k | \pi = 0.35) = 0.05$$



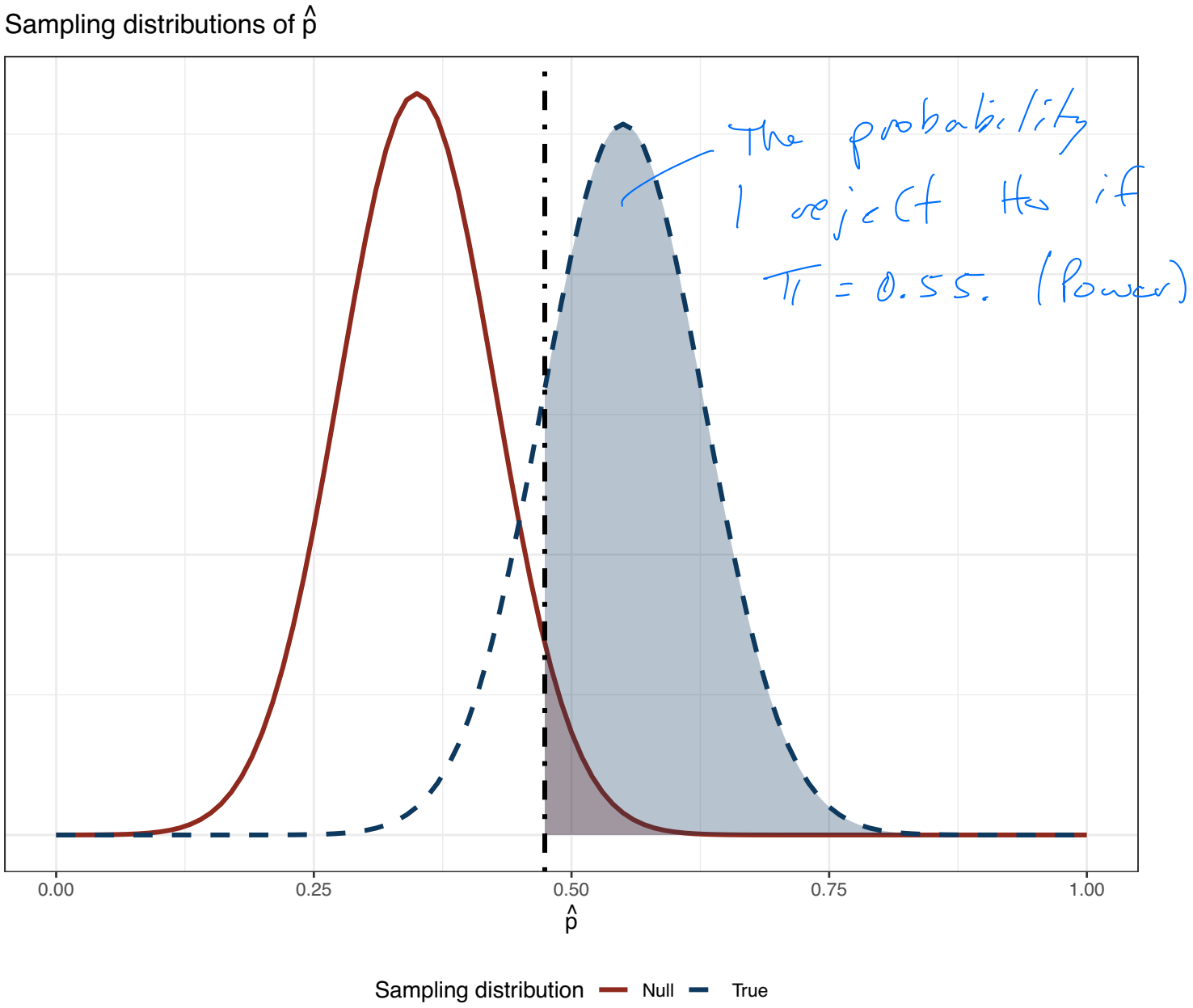
establishes a rule:
reject H_0 if $\hat{p} >$ approx. .45

Power: what is the probability I actually observe a $\hat{p} > .45$?

What is the probability that I get a sample proportion to the right of the vertical line **based on the actual, true value of** π?



The blue area is the power of this hypothesis test, which is 0.8329



Questions

1. What is the probability that I make a type I error?

It is α.

2. What happens to power as each of the following increase?

- n
- α
- π

- n: sample size - as n incr, power also incr.
- α: significance level - as α decr, power decr.
- π: true value of π - as the distance between π & π_0 incr, power incr.

$P(\text{Type I}) = \alpha$

Power $= \beta$

$P(\text{Type II}) = 1 - \beta$

Examples

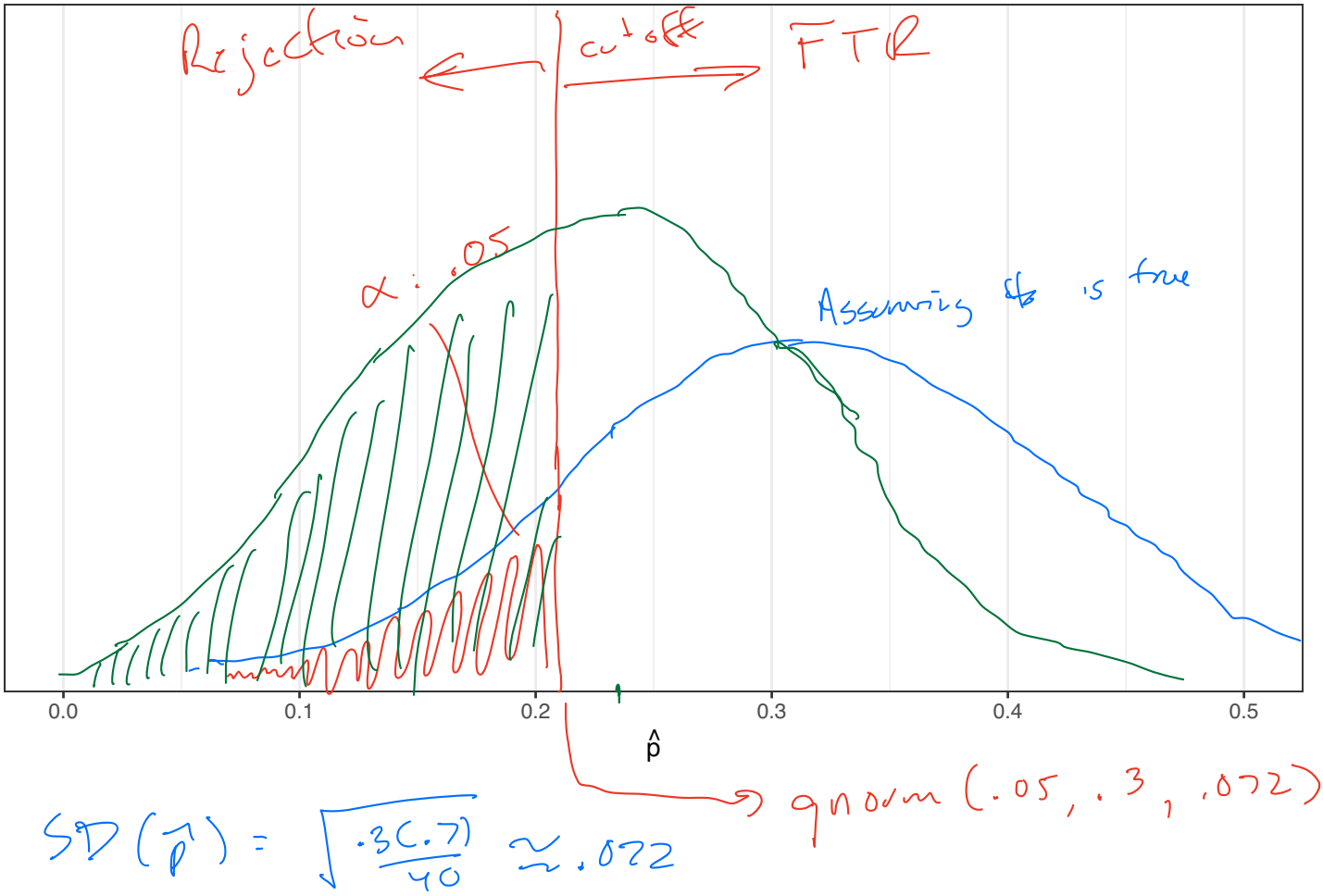
Researchers in Montana are interested in determining whether the proportion of brown trout in the Gallatin River has decreased since the last electrofishing event on the Gallatin, when it was determined that 30% of trout in the Gallatin River are brown trout. **State the null and alternative hypotheses in words and symbols.**

$H_0: \pi = .3$

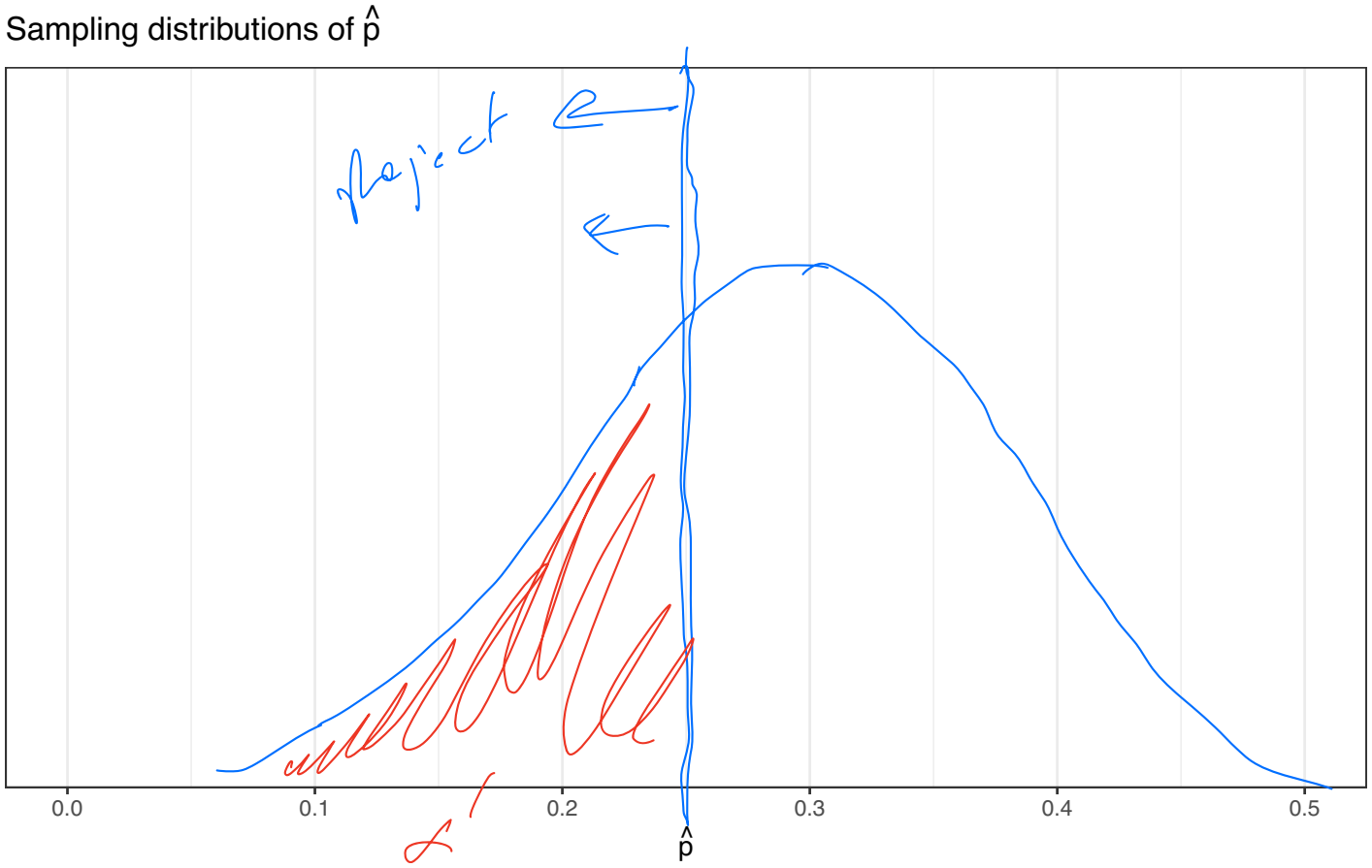
$H_a: \pi < .3$

Suppose you have good reason to believe the true proportion of brown trout in the Gallatin River is actually 0.24. If researchers take a sample of 40 trout from the Gallatin, what is the probability they will be able to detect that the null hypothesis is false at the $\alpha = 0.05$ level?

Sampling distributions of $\hat{p}$



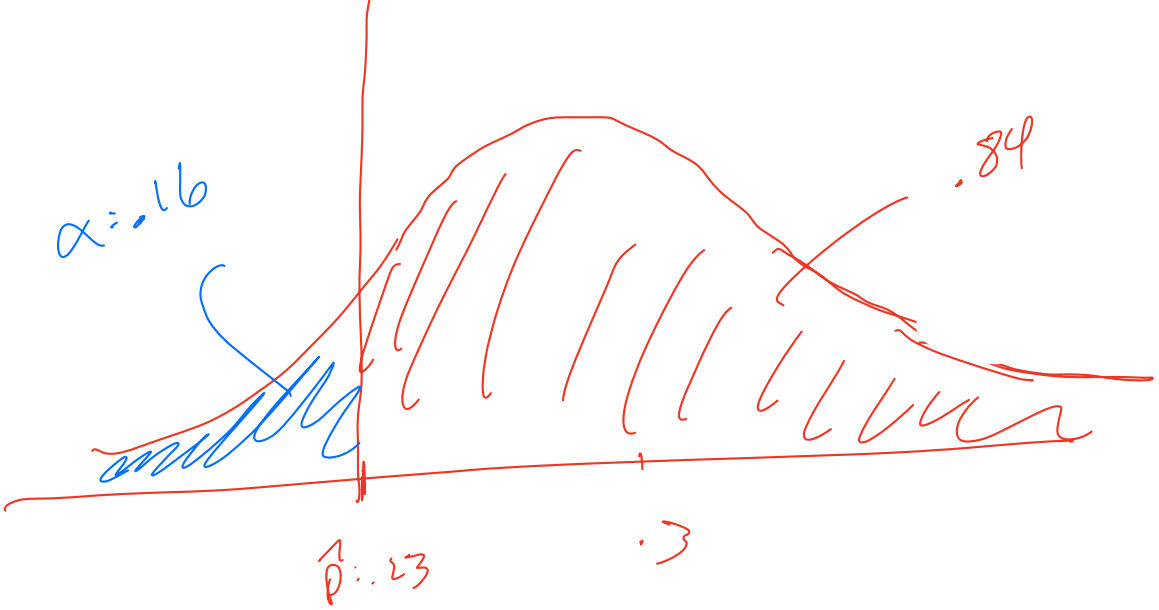
Again suppose you have good reason to believe the true proportion of brown trout in the Gallatin River is actually 0.24. If researchers take a sample of 40 trout from the Gallatin, what is the probability they will be able to detect that the null hypothesis is false if they reject the null hypothesis is $\hat{p} < .25$? What is the significance level of such a test?



$H_0: \pi = .3$ $n = 40$ s.t. $SD(\hat{p}) = .07$

$H_a: \pi < .3$

Rejecting rule: $\hat{p} < .23$


α = .16
.84
$\hat{p}$ = .23
.3